Sampling Distributions Free Response

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).**

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3. Trains carry bauxite ore from a mine in Canada to an aluminum processing plant in northern New York state in hopper cars. Filling equipment is used to load ore into the hopper cars. When functioning properly, the actual weights of ore loaded into each car by the filling equipment at the mine are approximately normally distributed with a mean of 70 tons and a standard deviation of 0.9 ton. If the mean is greater than 70 tons, the loading mechanism is overfilling.

(a) If the filling equipment is functioning properly, what is the probability that the weight of the ore in a randomly selected car will be 70.7 tons or more? Show your work.

(b) Suppose that the weight of ore in a randomly selected car is 70.7 tons. Would that fact make you suspect that the loading mechanism is overfilling the cars? Justify your answer.

(c) If the filling equipment is functioning properly, what is the probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more? Show your work.

(d) Based on your answer in part (c), if a random sample of 10 cars had a mean ore weight of 70.7 tons, would you suspect that the loading mechanism was overfilling the cars? Justify your answer.

Question 2

Qualification: AP Statistics

Areas: Probability, Sampling Distributions

Subtopics: Sampling Distribution of xbar, Normal Distribution - Finding probabilities (normcd), Normal Distribution - Two distributions (combining variables), At Least Probability Calculation

Paper: Part-A / Series: 2006 / Difficulty: Somewhat Challenging / Question Number: 3

3. The depth from the surface of Earth to a refracting layer beneath the surface can be estimated using methods developed by seismologists. One method is based on the time required for vibrations to travel from a distant explosion to a receiving point. The depth measurement (M) is the sum of the true depth (D) and the random measurement error (E). That is, $M = D + E$. The measurement error (E) is assumed to be normally distributed with mean 0 feet and standard deviation 1.5 feet.

(a) If the true depth at a certain point is 2 feet, what is the probability that the depth measurement will be negative?

(b) Suppose three independent depth measurements are taken at the point where the true depth is 2 feet. What is the probability that <u>at least one</u> of these measurements will be negative?

(c) What is the probability that the <u>mean</u> of the three independent depth measurements taken at the point where the true depth is 2 feet will be negative?

Question 3

Qualification: AP Statistics

Areas: Probability, Sampling Distributions

Subtopics: Sampling Distribution of Sample Mean , Binomial Distribution, Normal Distribution - Finding probabilities (normcd), Percentile, Normal Distribution - Given probability, work backwards (inv norm), At Least Probability Calculation

Paper: Part-A / Series: 2006-Form-B / Difficulty: Somewhat Challenging / Question Number: 3

3. Golf balls must meet a set of five standards in order to be used in professional tournaments. One of these standards is distance traveled. When a ball is hit by a mechanical device, Iron Byron, with a 10-degree angle of launch, a backspin of 42 revolutions per second, and a ball velocity of 235 feet per second, the distance the ball travels may not exceed 291.2 yards. Manufacturers want to develop balls that will travel as close to the 291.2 yards as possible without exceeding that distance. A particular manufacturer has determined that the distances traveled for the balls it produces are normally distributed with a standard deviation of 2.8 yards. This manufacturer has a new process that allows it to set the mean distance the ball will travel.

   (a) If the manufacturer sets the mean distance traveled to be equal to 288 yards, what is the probability that a ball that is randomly selected for testing will travel too far?

   (b) Assume the mean distance traveled is 288 yards and that five balls are independently tested. What is the probability that <u>at least one</u> of the five balls will exceed the maximum distance of 291.2 yards?

   (c) If the manufacturer wants to be 99 percent certain that a randomly selected ball will not exceed the maximum distance of 291.2 yards, what is the largest <u>mean</u> that can be used in the manufacturing process?

3. Big Town Fisheries recently stocked a new lake in a city park with 2,000 fish of various sizes. The distribution of the lengths of these fish is approximately normal.

(a) Big Town Fisheries claims that the mean length of the fish is 8 inches. If the claim is true, which of the following would be more likely?

- A random sample of 15 fish having a mean length that is greater than 10 inches

  or

- A random sample of 50 fish having a mean length that is greater than 10 inches

  Justify your answer.

(b) Suppose the standard deviation of the sampling distribution of the sample mean for random samples of size 50 is 0.3 inch. If the mean length of the fish is 8 inches, use the normal distribution to compute the probability that a random sample of 50 fish will have a mean length less than 7.5 inches.

(c) Suppose the distribution of fish lengths in this lake was nonnormal but had the same mean and standard deviation. Would it still be appropriate to use the normal distribution to compute the probability in part (b) ? Justify your answer.

Question 5

2. The graph below displays the relative frequency distribution for X, the total number of dogs and cats owned per household, for the households in a large suburban area. For instance, 14 percent of the households own 2 of these pets.
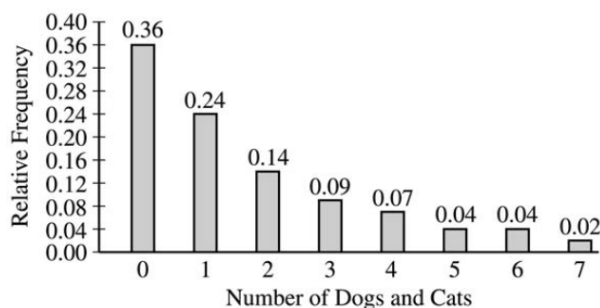
(a) According to a local law, each household in this area is prohibited from owning more than 3 of these pets. If a household in this area is selected at random, what is the probability that the selected household will be in violation of this law? Show your work.

(b) If 10 households in this area are selected at random, what is the probability that exactly 2 of them will be in violation of this law? Show your work.

(c) The mean and standard deviation of X are 1.65 and 1.851, respectively. Suppose 150 households in this area are to be selected at random and $\overline{X}$, the mean number of dogs and cats per household, is to be computed. Describe the sampling distribution of $\overline{X}$, including its shape, center, and spread.

2. Four different statistics have been proposed as estimators of a population parameter. To investigate the behavior of these estimators, 500 random samples are selected from a known population and each statistic is calculated for each sample. The true value of the population parameter is 75. The graphs below show the distribution of values for each statistic.
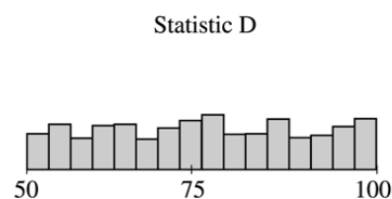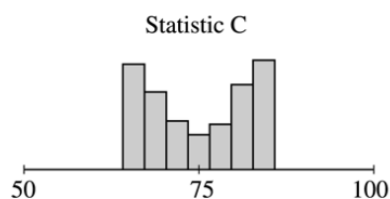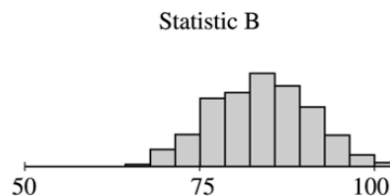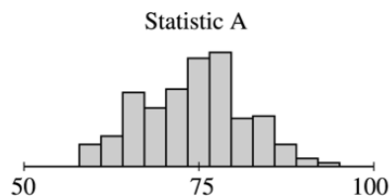
(a) Which of the statistics appear to be unbiased estimators of the population parameter? How can you tell?

(b) Which of statistics A or B would be a better estimator of the population parameter? Explain your choice.

(c) Which of statistics C or D would be a better estimator of the population parameter? Explain your choice.

Question 7

2. A tire manufacturer designed a new tread pattern for its all-weather tires. Repeated tests were conducted on cars of approximately the same weight traveling at 60 miles per hour. The tests showed that the new tread pattern enables the cars to stop completely in an average distance of 125 feet with a standard deviation of 6.5 feet and that the stopping distances are approximately normally distributed.

(a) What is the 70th percentile of the distribution of stopping distances?

(b) What is the probability that at least 2 cars out of 5 randomly selected cars in the study will stop in a distance that is greater than the distance calculated in part (a) ?

(c) What is the probability that a randomly selected sample of 5 cars in the study will have a mean stopping distance of at least 130 feet?

6. A consumer organization was concerned that an automobile manufacturer was misleading customers by overstating the average fuel efficiency (measured in miles per gallon, or mpg) of a particular car model. The model was advertised to get 27 mpg. To investigate, researchers selected a random sample of 10 cars of that model. Each car was then randomly assigned a different driver. Each car was driven for 5,000 miles, and the total fuel consumption was used to compute mpg for that car.

(a) Define the parameter of interest and state the null and alternative hypotheses the consumer organization is interested in testing.

One condition for conducting a one-sample t-test in this situation is that the mpg measurements for the population of cars of this model should be normally distributed. However, the boxplot and histogram shown below indicate that the distribution of the 10 sample values is skewed to the right.

Miles per Gallon
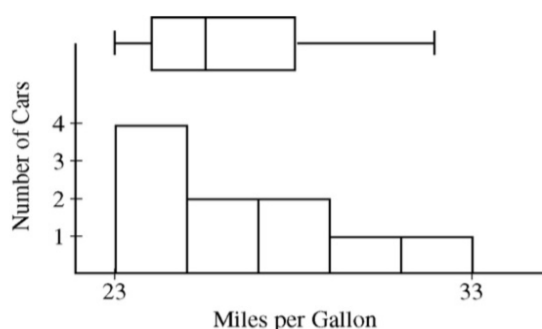
(b) One possible statistic that measures skewness is the ratio $\dfrac{\text{sample mean}}{\text{sample median}}$. What values of that statistic (small, large, close to one) might indicate that the population distribution of mpg values is skewed to the right? Explain.

(c) Even though the mpg values in the sample were skewed to the right, it is still possible that the population distribution of mpg values is normally distributed and that the skewness was due to sampling variability. To investigate, 100 samples, each of size 10, were taken from a normal distribution with the same mean and standard deviation as the original sample. For each of those 100 samples, the statistic $\dfrac{\text{sample mean}}{\text{sample median}}$ was calculated. A dotplot of the 100 simulated statistics is shown below.

sample mean/sample median
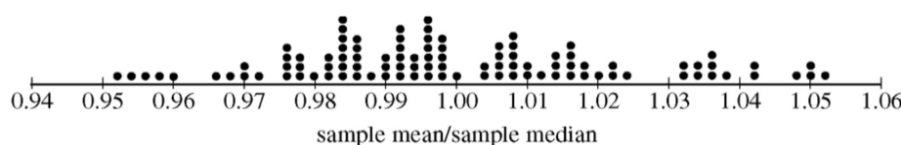
In the original sample, the value of the statistic $\dfrac{\text{sample mean}}{\text{sample median}}$ was 1.03. Based on the value of 1.03 and the dotplot above, is it plausible that the original sample of 10 cars came from a normal population, or do the simulated results suggest the original population is really skewed to the right? Explain.

(d) The table below shows summary statistics for mpg measurements for the original sample of 10 cars.

Minimum	Q1	Median	Q3	Maximum
23	24	25.5	28	32

Choosing only from the summary statistics in the table, define a formula for a different statistic that measures skewness.

What values of that statistic might indicate that the distribution is skewed to the right? Explain.

2. A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.

(a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.

(b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?

Question 10

Qualification: AP Statistics

Areas: Sampling Distributions , Data - Two Variable

Subtopics: Slope Interpretation, Sampling Distribution of xbar, Sample Mean, Linear Regression, Interpolation/Extrapolation

Paper: Part-B / Series: 2011-Form-B / Difficulty: Hard / Question Number: 6

6. Grass buffer strips are grassy areas that are planted between bodies of water and agricultural fields. These strips are designed to filter out sediment, organic material, nutrients, and chemicals carried in runoff water. The figure below shows a cross-sectional view of a grass buffer strip that has been planted along the side of a stream.
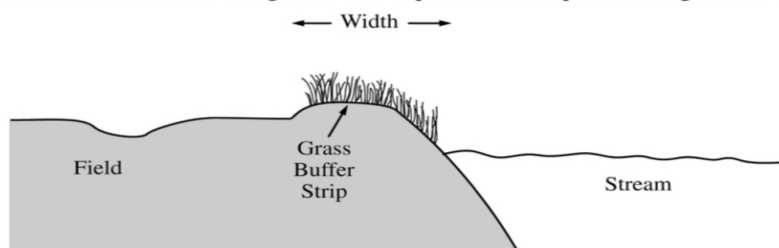
A study in Nebraska investigated the use of buffer strips of several widths between 5 feet and 15 feet. The study results indicated a linear relationship between the width of the grass strip (x), in feet, and the amount of nitrogen removed from the runoff water (y), in parts per hundred. The following model was estimated.

$$\hat{y} = 33.8 + 3.6x$$

(a) Interpret the slope of the regression line in the context of this question.

(b) Would you be willing to use this model to predict the amount of nitrogen removed for grass buffer strips with widths between 0 feet and 30 feet? Explain why or why not.

A scientist in California wants to know if there is a similar relationship in her area. To investigate this, she will place a grass buffer strip between a field and a nearby stream at each of eight different locations and measure the amount of nitrogen that the grass buffer strip removes, in parts per hundred, from runoff water at each location. Each of the eight locations can accommodate a buffer strip between 6 feet and 13 feet in width. The scientist wants to investigate which combination of widths will provide the best estimate of the slope of the regression line.

Suppose the scientist decides to use buffer strips of width 6 feet at each of four locations and buffer strips of width 13 feet at each of the other four locations. Assume the model, $\hat{y} = 33.8 + 3.6x$, estimated from the Nebraska study is the true regression line in California and the observations at the different locations are normally distributed with standard deviation of 5 parts per hundred.

(c) Describe the sampling distribution of the sample mean of the observations on the amount of nitrogen removed by the four buffer strips with widths of 6 feet.

(d) Using your result from part (c), show how to construct an interval that has probability 0.95 of containing the sample mean of the observations from four buffer strips with widths of 6 feet.

For the study plan being implemented by the scientist in California, the graph on the left below displays intervals that each have probability 0.95 of containing the sample mean of the four observations for buffer strips of width 6 feet and for buffer strips of width 13 feet. A second possible study plan would use buffer strips of width 8 feet at four of the eight locations and buffer strips of width 10 feet at the other four locations. Intervals that each have probability 0.95 of containing the mean of the four observations for buffer strips of width 8 feet and for buffer strips of width 10 feet, respectively, are shown in the graph on the right below.
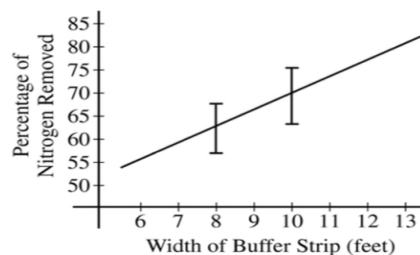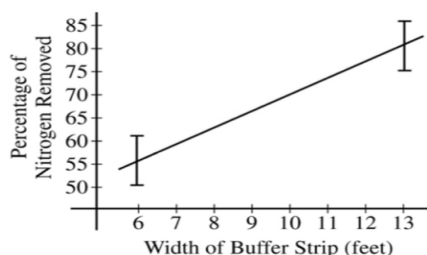
If data are collected for the first study plan, a sample mean will be computed for the four observations from buffer strips of width 6 feet and a second sample mean will be computed for the four observations from buffer strips of width 13 feet. The estimated regression line for those eight observations will pass through the two sample means. If data are collected for the second study plan, a similar method will be used.

(e) Use the plots above to determine which study plan, the first or the second, would provide a better estimator of the slope of the regression line. Explain your reasoning.

(f) The previous parts of this question used the assumption of a straight-line relationship between the width of the buffer strip and the amount of nitrogen that is removed, in parts per hundred. Although this assumption was motivated by prior experience, it may not be correct. Describe another way of choosing the widths of the buffer strips at eight locations that would enable the researchers to check the assumption of a straight-line relationship.

6. Two students at a large high school, Peter and Rania, wanted to estimate μ, the mean number of soft drinks that a student at their school consumes in a week. A complete roster of the names and genders for the 2,000 students at their school was available. Peter selected a simple random sample of 100 students. Rania, knowing that 60 percent of the students at the school are female, selected a simple random sample of 60 females and an independent simple random sample of 40 males. Both asked all of the students in their samples how many soft drinks they typically consume in a week.

(a) Describe a method Peter could have used to select a simple random sample of 100 students from the school.

Peter and Rania conducted their studies as described. Peter used the sample mean $\overline{X}$ as a point estimator for μ. Rania used $\overline{X}_{\text{overall}} = (0.6)\overline{X}_{\text{female}} + (0.4)\overline{X}_{\text{male}}$ as a point estimator for μ, where $\overline{X}_{\text{female}}$ is the mean of the sample of 60 females and $\overline{X}_{\text{male}}$ is the mean of the sample of 40 males.

Summary statistics for Peter's data are shown in the table below.

Variable	N	Mean	Standard Deviation
Number of soft drinks	100	5.32	4.13

(b) Based on the summary statistics, calculate the estimated standard deviation of the sampling distribution (sometimes called the standard error) of Peter's point estimator $\overline{X}$.

Summary statistics for Rania's data are shown in the table below.

Variable	Gender	N	Mean	Standard Deviation
Number of soft drinks	Female	60	2.90	1.80
	Male	40	7.45	2.22

(c) Based on the summary statistics, calculate the estimated standard deviation of the sampling distribution of Rania's point estimator $\overline{X}_{\text{overall}} = (0.6)\overline{X}_{\text{female}} + (0.4)\overline{X}_{\text{male}}$.

A dotplot of Peter's sample data is given below.

Number of Soft Drinks

Comparative dotplots of Rania's sample data are given below.

Number of Soft Drinks
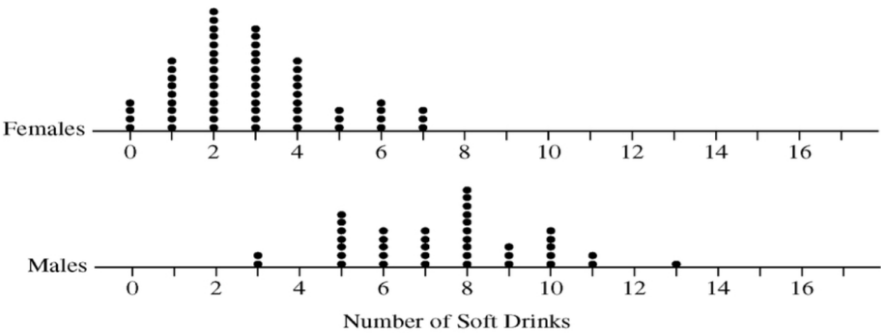
(d) Using the dotplots above, explain why Rania's point estimator has a smaller estimated standard deviation than the estimated standard deviation of Peter's point estimator.

Question 12

5. Psychologists interested in the relationship between meditation and health conducted a study with a random sample of 28 men who live in a large retirement community. Of the men in the sample, 11 reported that they participate in daily meditation and 17 reported that they do not participate in daily meditation.

The researchers wanted to perform a hypothesis test of

$$H_0 : p_m - p_c = 0$$
$$H_a : p_m - p_c < 0,$$

where p_m is the proportion of men with high blood pressure among all the men in the retirement community who participate in daily meditation and p_c is the proportion of men with high blood pressure among all the men in the retirement community who do not participate in daily meditation.

(a) If the study were to provide significant evidence against H_0 in favor of H_a, would it be reasonable for the psychologists to conclude that daily meditation causes a reduction in blood pressure for men in the retirement community? Explain why or why not.

The psychologists found that of the 11 men in the study who participate in daily meditation, 0 had high blood pressure. Of the 17 men who do not participate in daily meditation, 8 had high blood pressure.

(b) Let $\hat{p}_m$ represent the proportion of men with high blood pressure among those in a random sample of 11 who meditate daily, and let $\hat{p}_c$ represent the proportion of men with high blood pressure among those in a random sample of 17 who do not meditate daily. Why is it not reasonable to use a normal approximation for the sampling distribution of $\hat{p}_m - \hat{p}_c$?

Although a normal approximation cannot be used, it is possible to simulate the distribution of $\hat{p}_m - \hat{p}_c$. Under the assumption that the null hypothesis is true, 10,000 values of $\hat{p}_m - \hat{p}_c$ were simulated. The histogram below shows the results of the simulation.
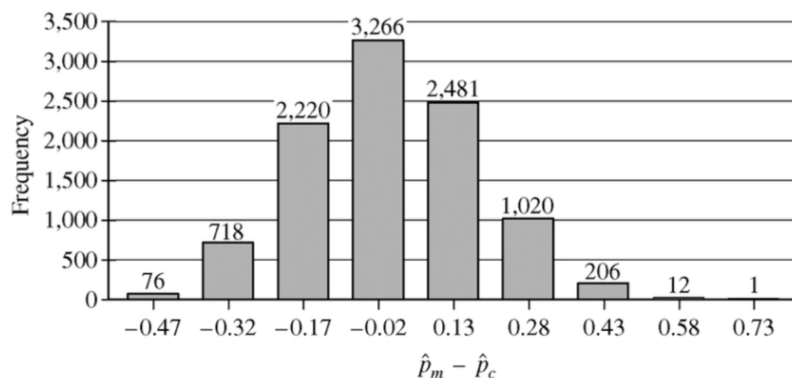
(c) Based on the results of the simulation, what can be concluded about the relationship between blood pressure and meditation among men in the retirement community?

3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

Question 14

Qualification: AP Statistics

Areas: Sampling Distributions

Subtopics: Generalize Conclusion to Population , Median vs. Mean, Percentile, Sampling Distribution Of Median

Paper: Part-B / Series: 2019 / Difficulty: Hard / Question Number: 6

6. Emma is moving to a large city and is investigating typical monthly rental prices of available one-bedroom apartments. She obtained a random sample of rental prices for 50 one-bedroom apartments taken from a Web site where people voluntarily list available apartments.

(a) Describe the population for which it is appropriate for Emma to generalize the results from her sample.

The distribution of the 50 rental prices of the available apartments is shown in the following histogram.
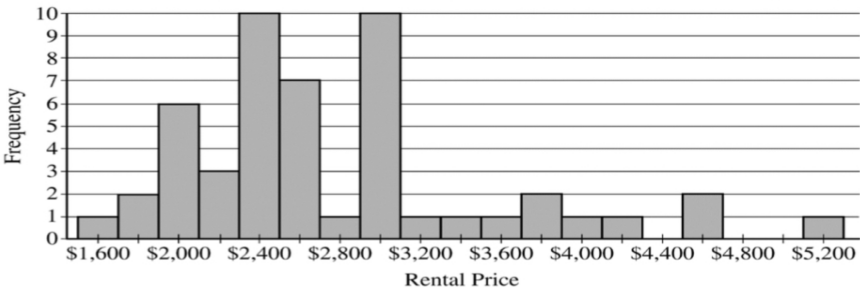
(b) Emma wants to estimate the typical rental price of a one-bedroom apartment in the city. Based on the distribution shown, what is a disadvantage of using the mean rather than the median as an estimate of the typical rental price?

(c) Instead of using the sample median as the point estimate for the population median, Emma wants to use an interval estimate. However, computing an interval estimate requires knowing the sampling distribution of the sample median for samples of size 50. Emma has one point, her sample median, in that sampling distribution. Using information about rental prices that are available on the Web site, describe how someone could develop a theoretical sampling distribution of the sample median for samples of size 50.

Because Emma does not have the resources to develop the theoretical sampling distribution, she estimates the sampling distribution of the sample median using a process called bootstrapping. In the bootstrapping process, a computer program performs the following steps.

- Take a random sample, with replacement, of size 50 from the original sample.
- Calculate and record the median of the sample.
- Repeat the process to obtain a total of 15,000 medians.

Emma ran the bootstrap process, and the following frequency table is the bootstrap distribution showing her results of generating 15,000 medians.

Bootstrap Distribution of Medians					
Median	Frequency	Median	Frequency	Median	Frequency
2,345	1	2,585	1	2,825	247
2,390	13	2,587.5	171	2,837.5	7
2,395	18	2,600	22	2,847.5	1
2,400	56	2,612.5	1,190	2,872.5	317
2,445	4	2,625	174	2,885	10
2,447.5	56	2,672.5	5	2,950	700
2,450	55	2,675	1,924	2,962.5	93
2,475	3	2,687.5	1,341	2,972.5	6
2,495	66	2,700	2,825	2,975	65
2,497.5	136	2,735	35	2,985	12
2,500	1,899	2,747.5	619	2,987.5	1
2,522.5	2	2,750	2	2,995	6
2,525	945	2,795	278	3,000	2
2,550	1,673	2,812.5	16	3,062.5	3

The bootstrap distribution provides an approximation of the sampling distribution of the sample median. A confidence interval for the median can be constructed using a percentage of the values in the middle of the bootstrap distribution.

(d) Use the frequency table to find the following.

(i) Value of the 5th percentile:

(ii) Value of the 95th percentile:

(e) Find the percentage of bootstrap medians in the table that are equal to or between the values found in part (d).

(f) Use your values from parts (d) and (e) to construct and interpret a confidence interval for the median rental price.

6. A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

   (a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.

The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

   (b) Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.

   (i) Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.

   (ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates 100,000 random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.

Graph I with $\sigma = 5$

Graph II with $\sigma = 8$

Graph III with $\sigma = 12$
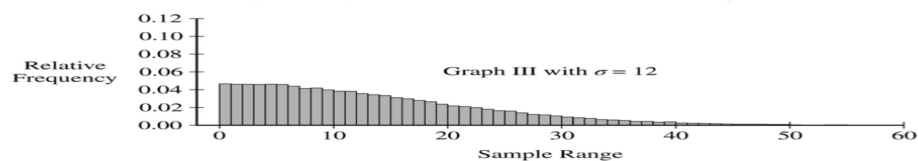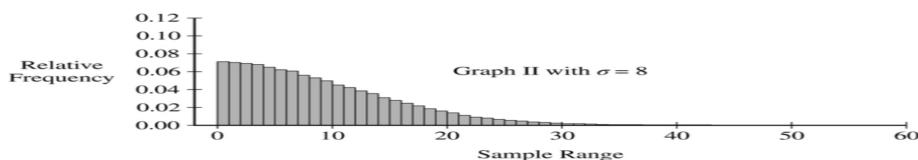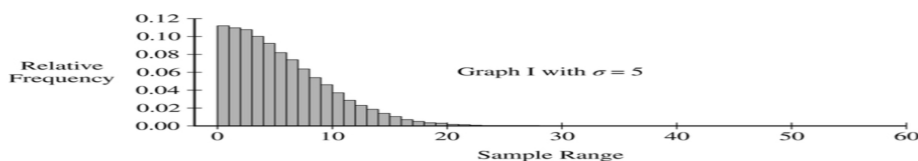
   (c) Use the information in the graphs to complete the following.

   (i) Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

   (ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg.

   (d) Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.

   Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.

   Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.